O-Level Mathematics

Notes

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LACAS

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Preface

This book has been designed for O Level students. It covers latest syllabus prescribed by CIE . I tried to make an attempt to present the material in a simple, clear and straightforward way.Three procedures are indispensable in the enjoyment and success in Mathematics: understanding, memorization and practice.The purpose of this book is to help the students in doing these. A large number of practice questions from past papers examination has been provided to help the students in learning of step-by-step procedure of problem solving. This will enable students to revise and practice systematically. Students are urged to grapple with these questions for acquiring solid understanding.

Thanks are due to the LACAS family for providing professionaI environment and Ms Nausheen Khan to assist in logical thinking. I acknowledge with thanks the generous cooperation by Mr Aamir Mustafa, Mr Asghar Hayat, Mr Qamar Fayyaz,Mr Zahid Amin and my son Junaid Rafique. I regret any error and misprints and pledge to correct these in my next edition.

Sincerely Yours

Rafique Akhtar Baloch

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Salient Features of the Book

1. Each topic consists of basic skills and comprehensive notes which are helpful to solve the questions.
2. All topics of syllabus are arranged in a manner which keep the learner's interest alive.
3. There is penalty of practice questions at the end of each topic.
4. Answers of all practice questions are given at the end, so that the students can easily analysis their performance.
5. Useful tips are given to solve questions in minimum time.
6. Some Practice questions are given with marks allocation
7. At the end formulae sheet is given for complete preparation

Dedicated to

“My family”

Contents

Numbers

Natural Number

Numbers which are used for counting purpose are called natural numbers Ex: 1, 2, 3, 4, ...100, ...

Whole Number:

Natural numbers including 0 are called whole numbers Exp: 0, 1, 2, 3, 4, ...

Integers:

Positive natural numbers negative natural numbers along with 0 are called integers.

Exp: -4, -3, -2,-1, 0, 1, 2, 3, 4, ...

Rational Numbers:

Numbers which are in the form of $\frac{p}{q}$ ($q \neq 0$) where p and q are positive or negative whole number are called as rational numbers. Exp : $\frac{1}{2}$, $\frac{3}{4}$, $\frac{-5}{7}$, $\frac{49}{-56}$, ...

Irrational Number:

Number like $\sqrt{2}$, π cannot be expressed as rational numbers. Such type of numbers are called as irrational numbers Exp : $\sqrt{5}, \sqrt{17}$,.....

Terminating Decimals:

These are decimals numbers which stop after a certain number of decimal places. for example, 7/8=0.875, is a terminating decimals because it stop (terminates) after 3 decimal places.

Recurring Decimals:

These are same decimal numbers which keep repeating a digit or group of digits : for example 137/259=0.528957528957528957..... is a recurring decimals, the six digits 528957 repeat in this order. Recurring decimals are written with dots over the first and last digit of the repeating digits, e.g 0.528957

- The order of operations follows the BODMAS rule-
  Brackets (Parentheses)
  of (Orders i.e Powers and square roots, Cube roots etc)
  Divide
  Multiply
  Add
  Subtract
- Even numbers: numbers which are exactly divisible by 2, e.g 0,2,4,6,8,.....
- Odd numbers: numbers which are not exactly divisible by 2, e.g; 1,3,5,7,.....
- Real Numbers are made of all possible ratioal and irrational numbers
- An integers is a whole number
- A prime number is divisible only by itself and by one(1). 1 is not a prime number. It has only two factors. 1 and the number itself.
- The exact value of rational number can be written down as the ratio of two whole numbers

- The exact value of an irrational number cannot be written down.
- A sequare number is the result of multiplying a number by itself.
  Exp: $1^2, 2^2, 3^2, \ldots\ldots\ldots\ldots\ldots\ldots$ i.e 1,4,9,.....
- A cube number is the result of multiplying a number by itself three times
  Ex: $1^3, 2^3, 3^3, \ldots\ldots\ldots\ldots\ldots\ldots$ i.e 1,8,27,.....
- The factors of a number are the numbers which divide exactly into two
  Exp: Factor of 36 are
  1,2,3,4,6,9,12,18,36
- Multiples of a number are the numbers in its times table
  Exp: multiples of 6 are 6,12,18,24,30.....

Significant figures :

Examples:

8064=8000 (correct to 1 significant figures)
8064=8100 (correct to 2 significant figures)
8064=8060 (correct to 3 significant figures)
0.00508=0.005 (correct to 1 significant figures)
0.00508=0.0051(correct to 2 significant figures)
2.00508=2.01 (correct to 3 significant figures)

Decimal Places

Examples:

0.0647=0.1 (correct to 1 Decimal places)
0.0647=0.06(correct to 2 Decimal places)
0.0647=0.065 (correct to 3 Decimal places)
2.0647=2.065 (correct to 3 Decimal places)

Summary

1 To write the different numbers in ascending or descending orde, first change all given numbers in decimal form then write it in required order by replacing them in original given form.

2 To find the smallest integer value of n in given multiple. First change them into product of prime number and then compare all prime factors in given multiple. The product of missing numbers is required value of n. Exp: smallest interger value of n for which 54n is the multiple of 14.
54n=2x3x3x3xn, 14 =2x7=>n=7

3 **Prime number:**the number which are only divisible by itself starting from 2
Exp: 2,3,5,7,11,.....

4 **Square numbers:**when a number is multiplied by itself two time. Then its answer is called square number or the number when its square roots is in whole number
Exp 2x2=4 , 3x3=9, 7x7=49,4,9,49 are square number

5 **Cube Numbers**
Cube roots of any number is whole number or answer of number, when multiplied it 3 times
To find value of unknown variable, write each term with power of 3. Product of missing number is answer.

6 **Rational numbers:**The number which can be written into the fraction of whole number with non zero denominator.
Exp $2, \frac{2}{3}, \frac{5}{4}$ etc

7 **Irrational Numbers:** all numbers in root whose answer is not a whole number and π is also irrational number but $\frac{22}{7}$ is ratinal number
Exp: $\sqrt{2}, \pi, \sqrt{\frac{7}{3}}$ etc

8 Largest or maximum integer which is factor or highest common factor means HCF and smallest multiple or minimum interger which is multiple or lowest common multiples mean LCM,

9 Significant figure means number of non- zero digits counted from left to right. if nearest right digit to the last digit of answer is 5 or more then 5 then add 1 in last digit other wise remain same, and put zero in the place of eleminated digits before decimal.

10 Decimal place means number of digits count after decimal from left to right.
Exp: 2537.43, 1843.56

One significant figure	Two significant figure	One Deciml Places
3000.	2500.	2537.4
2000.	1800.1843.6	

In Index Form

11 H.C.F

Product of only common terms with smallest power

12 L.C.M

Product of common terms with largest power and all non-common terms

To find value of unknown variable in **HCF or LCM** use comparison method for power of terms with prime factors.

13 Composite Number

Any number divisible by any other number than itself or by 1.

14 Rounding off

I. **Decimal places**

Number of digits space counted from left to right after decimal

II. **Significant figures**

No of non- zero digits counted form left to right

- Before decimal make all ignore digits zero
- After decimal don't put zero for ignore digits

15 Estimation

Round off each number, such that its simplification is possible

Practice Questions

Paper - 1

1 (a) Evaluate $12+8\div(9-5)$. [1]

(b) Evaluate $0.018\div0.06$. [1]

2 (a) Evaluate $3\frac{1}{4}-1\frac{4}{5}$. [1]

(b) Evaluate 3.01×0.02. [1]

3 (a) Evaluate $\frac{1}{7}+\frac{3}{4}$. [1]

(b) Evaluate $5\frac{1}{3}\div1\frac{3}{5}$. [2]

4 (a) Evaluate $\frac{1.3+2.9}{0.2}$. [1]

(b) Evaluate $2\frac{1}{4}\times\frac{1}{5}$. [1]

5 (a) Evaluate $8+2\times1.3$. [1]

(b) Express 0.06 as a fraction, [1]

6 (a) Evaluate $\frac{2}{5}+\frac{3}{8}$. [1]

(b) Evaluate $1\frac{2}{3}\times2\frac{1}{4}$, [1]

7 (a) Evaluate $12+6\div2-8$. [1]

(b) Evaluate 2.6×0.2. [1]

8 Evaluate

(a) $1.5-0.2\times4$, [1]

(b) $4.2\div0.07$. [1]

9 (a) Evaluate $6.3\div0.09$. [1]

(b) Find the decimal number that is exactly halfway between 3.8 and 4.3 [1]

10 Evaluate

(a) $\frac{1}{2}-\frac{3}{7}$, [1]

(b) $2\frac{2}{3}\times1\frac{3}{4}$. [1]

11 Evaluate

(a) $25-18.3$, [1]

(b) 1.7×0.03. [1]

12 It is given that $\frac{2}{3}=\frac{8}{b}$ and $\frac{8}{b}=\frac{n}{39}$ are equivalent fractions. [1]

Find the value of d and the value of n. [1]

13 Evaluate [1]

(a) $10-7.56$, [1]

(b) 0.105×0.2. [1]

14 (a) Evaluate $63\div0.9$. [1]

(b) Add brackets to the expression in the answer space to make it correct. [1]

$$1 + 72 \div 4 \times 2 = 10$$

15 Express as a single fraction in its lowest terms,

(a) $\frac{8}{9}\times\frac{3}{4}$, [1]

(b) $\frac{3}{4}-\frac{2}{3}$. [1]

16 (a) Evaluate $\frac{4}{9}+\frac{2}{5}$. [1]

(b) Evaluate $1+0.6\div0.02$. [1]

17 (a) Evaluate $3\times1\frac{4}{7}$. [1]

18 (b) Evaluate 1.3×0.3. [1]

(a) Evaluate 0.03×0.3. [1]

(b) Evaluate $5-2(3-1.4)$. [1]

19 (a) Evaluate $12+8\div(9-5)$. [1]

(b) Evaluate $0.018\div0.06$. [1]

20 (a) Evaluate $(2.05+1.4)\times0.2$. [1]

(b) Evaluate $12-6\div3+4$. [1]

21 Arrange these numbers in order, starting with the smallest.

$\frac{3}{4}$ 0 -1 $-\frac{17}{20}$ $-\frac{4}{5}$ [2]

22 Write these numbers in order of size, starting with the smallest.

$\frac{13}{20}$ 0.7 $\frac{7}{12}$ 0.64 $\frac{5}{8}$ [2]

23 Write these numbers in order of size, starting with the smallest. [2]

$\frac{1}{3}$ 0.32 $\frac{15}{40}$ 0.3 $\frac{9}{31}$

24 Write these numbers in order of size, starting with the smallest. [2]

2.1×10^{-3} 4.2×10^{-4} 1.7×10^{-5} 3.5×10^{-4}

25 Write these values in order, starting with the smallest.

$\frac{7}{200}$ 4% $\frac{3}{50}$ 0.03 $\frac{1}{20}$ [2]

26 Write these numbers in order of size, starting with the smallest.

$\sqrt{17}$ 4 4.5 $\sqrt[3]{63}$ [2]

27 Write these values in order, starting with the smallest.

$\frac{1}{30}$ 0.03 $\frac{1}{10}$ 5% $\frac{2}{25}$ [2]

28 (a) (i) Express 7056 as the product of its prime facto [2]

(ii) Hence evaluate $\sqrt{7056}$. [1]

(b) $\sqrt{5\frac{1}{16}}$ can be expressed as the rational number $\frac{p}{q}$

Find the value of p and the value of q. [1]

(c) Write down an example of an irrational number. . [1]

29 (a) Express 154 as the product of its prime factors. .[1]

(b) Find the lowest common multiple of 154 and 49. .[1]

30 The numbers 294 and 784, written as

$294 = 2\times3\times7^2$, $784 = 2^4\times7^2$.

Find

(a) the largest integer which is a factor of both 294 and 784, ..[1]

(b) $\sqrt{784}$. ..[1]

31 Written as a product of prime factors, $168 = 2^3\times3\times7$. .[1]

(a) Express 140 as a product of its prime factors. .[1]

(b) Find the highest common factor of 168 and 140.

(c) Find the smallest positive integer, n, such that $168n$ is a square number. .[1]

32 (a) Express 108 as a product of its prime factor . [1]

(b) Written as products of their prime factors,

$N = 2^p\times5^q\times7^r$ and $500 = 2^2\times5^3$.

The highest common factor of N and 500 is $2^2\times5^2$.

The lowest common multiple of N and 500 is $2^3\times5^3\times7$. . [2]

Find p, q and r.

33 Written as the product of its prime factors, $360 = 2^3\times3^2\times5$.

(a) Write 108 as the product of its prime factors. ..[1]

(b) Find the lowest common multiple of 108 and 360. Give your answer as the product of its prime factors. ..[1]

(c) Find the smallest positive integer k such that $360k$ is a cube number. ..[1]

34 (a) Express 198 as the product of its prime factors.

(b) $M = 2^2\times3\times5^2$ $N = 2^3\times3^2\times7$

(i) Find the largest number that divides exactly into M and N. [1]

(ii) Find the smallest value of k, such that $M\times k$ is a cube number. [1]

35 **A number written as the product of its prime factors is $2^2\times5^2\times7$.**

(a) Evaluate this number. [1]

(b) The lowest common multiple of $2^2\times5^2\times7$ and another number, N, is $2^2\times3\times5^2\times7^2$.

Find the lowest possible value of N. [1]

36 (a) Express 96 as a product of its prime factors. [1]

(b) 24 is a common factor of 96 and the integer n. [1]

37 (a) Express 500 as the product of its prime factors.

(b) $M = 2\times3^2$ $N = 2^4\times3^2$

Find the values of p and q when

(i) $M\times N = 2^p\times3^q$, [1]

(ii) $M\div N = 2^p\times3^q$, [1]

(iii) $N^2 = 2^p\times3^q$ [1]

38 (a) Express 1200 as the product of its prime factor [1]

(b) Find the smallest value of n, such that $120n$ is a square number. [1]

39 (a) Write 168 as a product of its prime factors. [2]

(b) The highest common factor of 168 and N is 42. [2]

Given that $200 < N < 300$, find the two possible values of N.

40 $p = 2^3 \times 3 \times 5^2 \qquad q = 2 \times 3^2 \times 5$

(a) Find the highest common factor (HCF) of p and q. [1]

(b) Find the lowest common multiple (LCM) of p, q and 21.
Give your answer as the product of prime factors. [1]

(c) Find the smallest integer N, such that pN is a square number. [1]

41 (a) James thinks of a **two-digit** number. It is a cube number.
It is an even number. What is his number? [1]

(b) Omar thinks of a **two-digit** number. It is a factor of 78.
It is a prime number. What is his number? [1]

(c) Write down an irrational number between 1 and 2. [1]

42 (a) Write down the two cube numbers between 10 and 100. ..[1]

(b) Write down the two prime numbers between 30 and 40. ..[1]

43 (a) Express 60 as a product of its prime factors. [1]

(b) Find the smallest possible integer m such that $60m$ is a square number. [1]

(c) The lowest number that is a multiple of both 60 and the integer n is 180 . [1]

Find the smallest possible value of n.

44

6	9	1

The three cards above can be rearranged to make three-digit numbers, for example 916.

Arrange the three cards to make [1]

(a) the three-digit number that is closest to 650,

(b) the three-digit number that is a multiple of 7, [1]

(c) a three-digit number that is a square number. [1]

45 (a) Write down an irrational number which has a value between 4 and 5. [1]

(b) Kofi is using number cards to form a 5-digit number.
His number is a multiple of 8. [1]
Complete the final digit of his number.

1	2	3	4	

46 (a) Write down an irrational number which has a value between 4 and 5. [1]

(b) Kofi is using number cards to form a 5-digit number.
His number is a multiple of 8. [1]
Complete the final digit of his number.

1	2	3	4	

47 Find the fraction that lies exactly halfway between $\frac{3}{5}$ and $\frac{3}{4}$. [2]

48 $\sqrt{35}$ $\sqrt{36}$ 36 $\frac{36}{37}$ 37 $\frac{37}{36}$ 3.7 [1]

From this list of numbers, write down [1]

(a) a prime number, (b) a square number, (c) an irrational number. [1]

49 It is given that $68.2 \times 0.235 = 16.027$. Hence evaluate

(a) 0.0682×2350, ..[1]

(b) $160.27 \div 0.0235$. [1]

50

(a) Evaluate $6 \times 3 + 8 \div 2$. [1]

(b) By writing each number correct to 1 significant figure, estimate the value of .[2]

$$\frac{19.2 \times 9.09}{0.583}.$$

51 (a) Write 405 917 628 correct to three significant figures. [1]

(b) By writing each number correct to one significant figure, estimate the value of .[2]

$$\frac{41.3}{9.79 \times 0.765}.$$

52 (a) Express 0.047 852 correct to two decimal places. *Answer* .....[1]

(b) Estimate the value of $\sqrt{200}$, giving your answer correct to two significant figures.

Answer .....[1]

(c) By writing each number correct to one significant figure, estimate the value of

$$\frac{212 \times 1.97^2}{0.763}.$$ *Answer* .....[2]

53

(a) Write 0.004 075 1 correct to two significant figures.

Answer .....[1]

(b) $\sqrt{131}$ lies between two consecutive integers.
Complete the inequality below with these integers.

Answer ..... $< \sqrt{131} <$ .....[1]

(c) Add brackets to the statement below to make it correct.

$$3 \times 2 + 1^2 = 49$$ [1]

54 **By making suitable approximations,** estimate the value of $\frac{\sqrt{35.78} \times \sqrt[3]{1005}}{0.3012}$.
Show clearly the approximate values you use.

Answer ..... [2]

55 (a) Write the value of 1234.567, correct to 2 significant figures.

Answer ..... [1]

(b) **Write down** an estimate for the value of $\sqrt{\frac{28}{\pi}}$.

56 *Answer* ..... [1]

By writing each number correct to one significant figure, estimate the value of

$$\frac{29.3^2}{2.04 \times 0.874}.$$.[2]

57

By writing each number correct to 1 significant figure, calculate an estimate for the value of

$$\frac{614.2 \times 0.0304}{19.88}.$$.[2]

$120 = 2^3 \times 3 \times 5$

58 By writing each number correct to 2 significant figures, estimate the value of

$$\frac{1212.3}{299.35 \times \sqrt{24.73}}.$$ [2]

59 By writing each number correct to one significant figure, estimate the value of

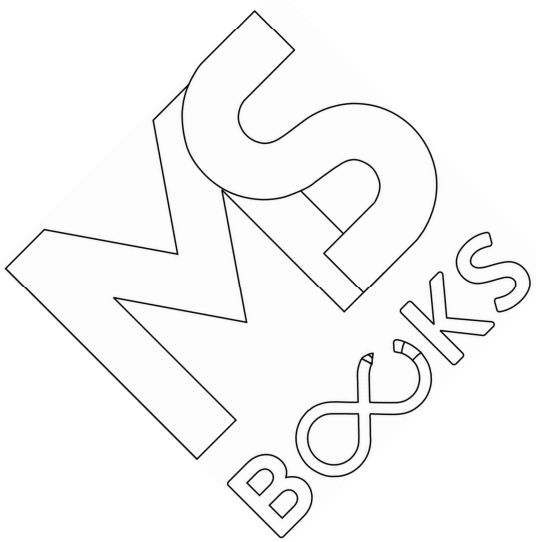
$$\frac{71.8 - 32.4}{0.198^2}.$$ [2]

60 By writing each number correct to 1 significant figure, estimate the value of

$$\frac{39.864 \times \sqrt{8.987}}{0.6013}.$$ [2]

61 By writing each number correct to 1 significant figure, estimate the value of

$$\frac{59.843^2}{20.13 \times 0.9024}.$$ [2]



Answers

1 (a) 14 (b) 0.3

2 (a) $1\frac{9}{20}$ (b) 0.0602

3 (a) $\frac{25}{28}$ (b) 2

4 (a) 21 (b) $\frac{9}{20}$

5 (a) 10.6 (b) $\frac{3}{50}$

6 (a) $\frac{31}{40}$ (b) $3\frac{3}{4}$

7 (a) 7 (b) 0.52

8 (a) 0.7 (b) 60

9 (a) 70 (b) 4.05

10 (a) $\frac{1}{14}$ (b) $4\frac{2}{3}$

11 (a) 6.7 (b) 0.051

12 d= 12 n= 26

13 (a) 2.44 (b) 0.021

14 (a) 70
(b) 1+[72÷(4x2)]= 10

15 (a) $\frac{2}{3}$ (b) $\frac{1}{12}$

16

(a)	$\frac{38}{45}$
(b)	31

17

(a)	$4\frac{5}{7}$ oe
(b)	[0.]39

18

(a)	0.009(
(b)	1.8

19 (a) 14 (b) 0.3(

20

(a)	0.69
(b)	(0).45(0

21 -1, $\frac{-17}{20}$, $\frac{-4}{5}$, 0, $\frac{3}{4}$

22 $\frac{7}{12}$ $\frac{5}{8}$ 0.64 $\frac{13}{20}$ 0.7

23 $\frac{9}{31}$ 0.3 0.32 $\frac{1}{3}$ $\frac{15}{40}$

24 1.7×10^{-5}, 3.5×10^{-4}, 4.2×10^{-4}, 2.1×10^{-3}

25 0.03 $\frac{7}{200}$ 4% $\frac{1}{20}$ $\frac{3}{50}$

26 $\sqrt[3]{63}$, 4, $\sqrt{17}$, 4.5

27 0.03 $\frac{1}{30}$ 5% $\frac{2}{25}$ $\frac{1}{10}$

28 (a) (i) $2^4 \times 3^2 \times 7^2$ (ii) 84
(b) ± 9, ± 4 (c) π

29 (a) 2 x 7 x 11 (b) $2 \times 7^2 \times 11$

30 (a) 98 (b) 28

31 (a) $2^2 \times 5 \times 7$ (b) 28 (c) 42

32 (a) $2^2 \times 3^2$ (b) p = 3, p= 2, r= 1

33 (a) $2^2 \times 3^3$ (b) $2^3 \times 3^3 \times 5$ (c) k= 75

34 (a) $2 \times 3^2 \times 11$ (b) (i) 12 (ii) 90

35

(a)	**700**
(b)	**147; or 3×7^2**

36

(a)	$2^5 \times 3$
(b)	72

37

(a)		$2^2 \times 5^3$
(b)	**(i)**	$p = 5$ and $q = 4$
	(ii)	$p = -3$ and $q = 0$
	(iii)	$p = 8$ and $q = 4$

38 (a) $2^4 \times 3 \times 5^2$ (b) 30

39

(a)	$2 \times 2 \times 2 \times 3 \times 7$ or $2^3 \times 3 \times 7$
(b)	210 and 294 only